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BRIEF COMMUNICATION

EFFECT OF THE NO-SLIP ASSUMPTION ON THE PREDICTION OF SOLID–LIQUID FLOW CHARACTERISTICS

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INTRODUCTION

A physical mechanistic model for the prediction of pressure drop and flow patterns for the flow of settling slurries in horizontal pipes was presented by Doron *et al.* (1987). The analysis was based on a two-layer model: a stationary or moving bed at the bottom of the pipe and a heterogeneous mixture of solid particles and carrier liquid at the upper part. One of the main assumptions underlying their analysis was the absence of slip between the solid and liquid phases. This assumption may seem quite unrealistic, especially regarding the bed layer, and may hamper the validity of the model. In order to examine the effect of the no-slip assumption on the model results, the bed layer is considered in the present work as a modified porous medium, where the velocities of the solids and the liquid in the bed are no longer identical. The results obtained by using the modified model show that the previous no-slip assumption for the bed layer is reasonable.

DORON *et al.* (1987) MODEL

We will first review briefly the two-layer model analysis of Doron *et al.* (1987). A stationary or moving bed is assumed to exist at the bottom of the pipe, with a heterogeneous mixture of solid particles and carrier liquid at the upper part (figure 1). The continuity equations for the two phases are:

$$U_h C_h A_h + U_b C_b A_b = U_s C_s A \qquad [1]$$

and

$$U_h(1 - C_h)A_h + U_b(1 - C_b)A_b = U_s(1 - C_s)A, \qquad [2]$$

where U_h and U_b are the mean velocities in the dispersed layer and in the bed, respectively, C_h and C_b are the mean volumetric concentrations in the two layers and A_h and A_b are the cross-sectional areas occupied by the two layers. Thus, the mixtures in the dispersed layer and in the bed are considered as two pseudo-fluids, so that the presence of the solid phase only modifies their effective properties. U_s is the slurry superficial velocity, C_s is the slurry input concentration and A is the pipe cross-sectional area.

Force balances on each layer yield the following two equations:

$$A_h \frac{dP}{dx} = -\tau_h S_h - \tau_i S_i \qquad [3]$$

and

$$A_b \frac{dP}{dx} = -F_b + \tau_i S_i, \qquad [4]$$

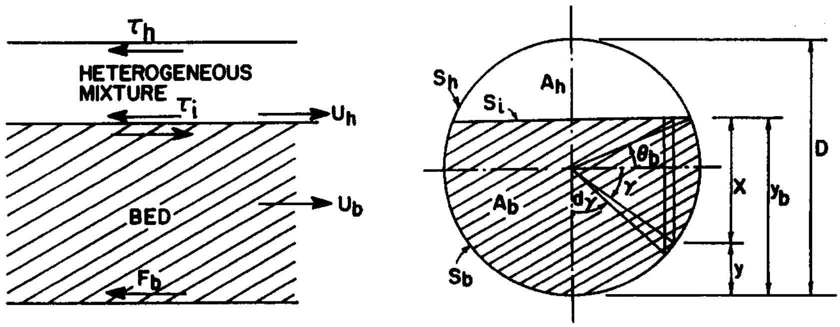

Figure 1. The two-layer model.

where dP/dx is the pressure drop and τ_h and τ_i are the upper layer shear stress and the interfacial shear stress acting on the perimeters S_h and S_i, respectively (figure 1). The shear stresses τ_h and τ_i are calculated using common constitutive expressions for smooth and rough pipe flow, respectively. F_b is the force acting on the bottom of the pipe which consists of two components: a dry friction force, F_{bd}, which is exerted by the solid particles in the bed on the surface of contact between the bed and the pipe wall calculated using a pseudo-hydrostatic pressure distribution; and a hydrodynamic resistance force, $\tau_b S_b$, which stems from the bed motion. τ_b is calculated in a similar way to τ_h. Note, that no force between the two phases exists in this formulation, since they are assumed to move at the same velocity within each layer.

The mechanism which governs the dispersion of the solid particles in the upper layer is represented by the well-known diffusion equation:

$$\epsilon \frac{d^2C}{dy^2} + w \frac{dC}{dy} = 0, \qquad [5]$$

where C is the volumetric concentration, y is the vertical coordinate (perpendicular to the pipe axis), ϵ is the diffusion coefficient and w is the particles' terminal settling velocity.

The Doron *et al.* (1987) model consists of the set of five equations, [1]–[5], which can be solved for any given set of physical properties of the two phases and operational conditions. The solution yields the following state variables: the pressure drop (dP/dx), the bed mean height (h_b), the mean concentration in the upper layer (C_h) and the mean velocities in the upper layer and in the bed (U_h and U_b, respectively). In addition, the concentration vertical distribution in the upper layer can be obtained.

MODIFIED MODEL

One of the fundamental assumptions of the Doron *et al.* (1987) model was the lack of relative axial motion between the solids and the liquid in each layer. This no-slip assumption seems quite reasonable for the upper layer, where the solid particles follow the liquid motion due to the relative dilution and high velocities (and hence turbulent mixing). However, in the bed layer the motion of the solid particles is impeded by enhanced wall friction (F_{bd}) as well as by interparticle forces. Thus, it seems more realistic to assume that the liquid within the bed will flow faster than the solid particles. Since the effect of relative velocity on the pressure drop associated with flow through porous media is very large, one could argue that even a small relative velocity might affect the flow characteristics considerably. Moreover, it is plausible that when the solid particles are stationary, liquid may seep through the solid bed (due to the existence of a downstream pressure gradient). The relaxation of the no-slip assumption regarding the bed layer underlies the modification of the model.

Continuity

Two continuity equations are now written for the two phases in the bed. Thus, the mean velocity in the bed, U_b, is replaced by the mean velocities of the solid particles and of the liquid in the bed, U_{bS} and U_{bL}, respectively.

The continuity equation for the solids is

$$U_h C_h A_h + U_{bs} C_b A_b = U_s C_s A, \quad [6]$$

where U_{bS} is the mean velocity of the solid particles in the bed.

The continuity equation for the liquid is

$$U_h(1 - C_h)A_h + U_{bL}(1 - C_b)A_b = U_s(1 - C_s)A, \quad [7]$$

where U_{bL} is the mean velocity of the liquid in the bed.

Momentum

Since the characteristics of the upper layer are unchanged, so is the form of the force balance for this layer, and it is represented by [3].

However, the expression for the interfacial shear stress, τ_i (which appears in [3]), has to be modified, since it incorporates the relative velocity between the two layers. In the present case, there is no "mean bed velocity" but rather two velocities associated with the two phases. The liquid flowing above the uppermost stratum of bed solid particles is considered as part of the upper layer, and its only contact with the bed is with those particles. Thus, the interfacial shear stress is based on the relative velocity of the upper layer with respect to the solids in the bed. Hence,

$$\tau_i = \tfrac{1}{2} f_i \rho_h |U_h - U_{bS}|(U_h - U_{bS}), \quad [8]$$

where f_i is the interfacial shear coefficient and $\rho_h = C_h \rho_S + (1 - C_h)\rho_L$; ρ_S and ρ_L are the densities of the solids and the liquid, respectively.

The force balance on the bed is also similar in form to the one used before, [4]. Whereas the mode of calculation of the dry friction force, F_{bd}, is not affected by the slip between the solids and the liquid, the hydraulic shear component, $\tau_b S_b$, of the force acting on the bottom of the pipe, F_b, has to be modified. The solids are assumed to be spherical rigid particles, hence their total area of contact with the pipe wall is negligible, and they contribute only Coulombic friction. The wall hydraulic friction arises from the motion of the liquid only, thus

$$\tau_b = \tfrac{1}{2} f_b \rho_L |U_{bL}| U_{bL}. \quad [9]$$

Diffusion

The diffusion equation is identical to the previous analysis, [5], since it is based on the behavior of the flow in the upper layer. The only modification is in the calculation of ϵ, which incorporates the interfacial shear stress, τ_i (see Doron *et al.* 1987).

Ergun equation

The existence of a relative velocity between the two phases in the bed creates an additional unknown in the model, hence an additional equation is needed. Consideration of the bed as a porous medium provides the additional equation which is required in order to "close" the model. The solid particles are viewed as forming a porous matrix and the liquid flows through the pores. It is assumed that commonly used expressions for the flow through porous media can be applied inside the bed layer. Note, that in this case the matrix itself may be moving, thus the relative velocity between the two phases should be inserted in these expressions.

The pressure drop for flow through porous media is most commonly represented by Darcy's law. For the case of packed beds of "large" spheres the pressure drop is given by an Ergun-type equation:

$$-\frac{dP}{dx} = B_1 u + B_2 u^2, \quad [10]$$

where u is the superficial velocity. In our case, it is $u = (1 - C_b)(U_{bL} - U_{bS})$. Numerous expressions for the constants have been proposed. The most frequently used are the Blake–Kozeny and the Burke–Plummer relationships:

$$B_1 = \frac{150 \mu C_b^2}{d_p^2 (1 - C_b)^3} \quad [11a]$$

and

$$B_2 = \frac{1.75\rho_L C_b}{d_p(1 - C_b)^3}, \qquad [11b]$$

respectively, where μ is the viscosity of the liquid and d_p is the solid particle diameter. Gibilaro *et al.* (1985) proposed the following correlations:

$$B_1 = \frac{17.3\mu C_b}{d_p^2(1 - C_b)^{4.8}} \qquad [12a]$$

and

$$B_2 = \frac{0.336\rho_L C_b}{d_p(1 - C_b)^{4.8}}. \qquad [12b]$$

The modified model is composed of a set of six equations, [3]–[7] and [10], which can be solved to yield the pressure drop, the bed height, the mean concentration in the upper layer, the mean velocity in the upper layer and the velocities of the two phases in the bed layer.

The modified model should reduce to the previous set of equations, [1]–[5], at the limit of $U_{bL} = U_{bS} = U_b$. This is indeed the case regarding all the modified equations and constitutive relations, except for the expression for τ_b, [9]. In the previous model it was assumed that the bed consists of a pseudo-fluid of apparent density ρ_b, where

$$\rho_b = C_b\rho_S + (1 - C_b)\rho_L \qquad [13]$$

and

$$\tau_b = \tfrac{1}{2}f_b\rho_b|U_b|U_b. \qquad [14]$$

It is suggested here, that the same reasoning that led to [9] should be applicable also to the previous model of Doron *et al.* (1987). In that case, too, the solid particles were assumed to contribute Coulombic fraction. Hence, the hydraulic shear component should be based on the flow of the liquid only, and ρ_b should be replaced by ρ_L in [14].

RESULTS

The modified model has been employed to compute the flow characteristics for various sets of operational conditions. The transition velocity between flow with a moving bed and flow with a stationary bed was examined, as well as the pressure gradients and the bed height. It has been found that the no-slip assumption has negligible effect on the results.

Consider the transition velocity between flow with a stationary bed and flow with a moving bed. According to the previous model, the mean velocity of the bed is attributed to the two phases, thus in order for a stationary bed to occur, the velocities of both the solids and the liquid in the bed layer must be zero. Since the modified model allows for slip between the two phases in the bed, it is possible for the solid particles to be at rest while liquid seeps through the porous bed. Such a situation is considered a stationary bed, since it is defined by the immobility of the solids. However, the averaged mean bed velocity is not zero, hence according to the previous model this case is considered as flow with a moving bed. Therefore, the transition between the two flow patterns can be considered as an important test case for the effect of the no-slip assumption. Implementation of the modified model yields predictions, which are virtually identical to those of the previous model (since the magnitudes of the differences between the results of the previous model and the modified one are fractions of a percent, we do not present a graphic comparison). This has been verified for various sets of operational conditions.

Figure 2 presents the pressure drop obtained for flow with a moving bed from the previous model (——) and from the modified model (----). The lines almost coincide, with a very minute deviation at the lower flow rates, where the modified model predicts slightly higher pressure gradients. This is due to the small increase in the bed heights as predicted by the modified model (figure 3). Similar results are obtained for the case where a stationary bed and a moving bed exist, as is demonstrated in figure 4.

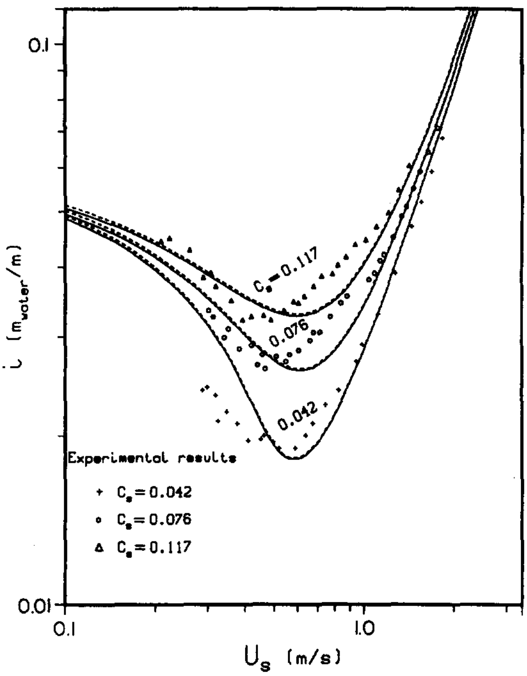

Figure 2. Dependence of pressure drop on slurry superficial velocity, flow with a moving bed, $\rho_S = 1240\ \mathrm{kg/m^3}$, $\rho_L = 1000\ \mathrm{kg/m^3}$, $d_p = 3$ mm, $D = 50$ mm; ——, original model; – – –, modified model.

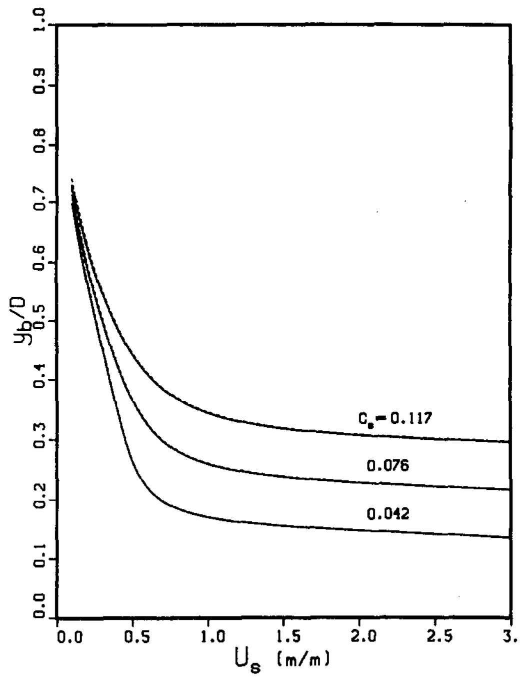

Figure 3. Dependence of bed height on slurry superficial velocity, $\rho_S = 1240\ \mathrm{kg/m^3}$, $\rho_L = 1000\ \mathrm{kg/m^3}$, $d_p = 3$ mm, $D = 50$ mm; ——, original model; – – –, modified model.

The results also are not affected by the choice of porous-medium correlation since the constants ([11a,b] and [12a,b]) assume similar values for maximum packing concentrations. Moreover, an artificial division of the constants B_1 and B_2 by a factor of 20 yields deviations of only up to approx. 10% in the results (figure 5). Note, that for large values of the constants B_1 and B_2 the limit of no-slip (the previous model) is approached.

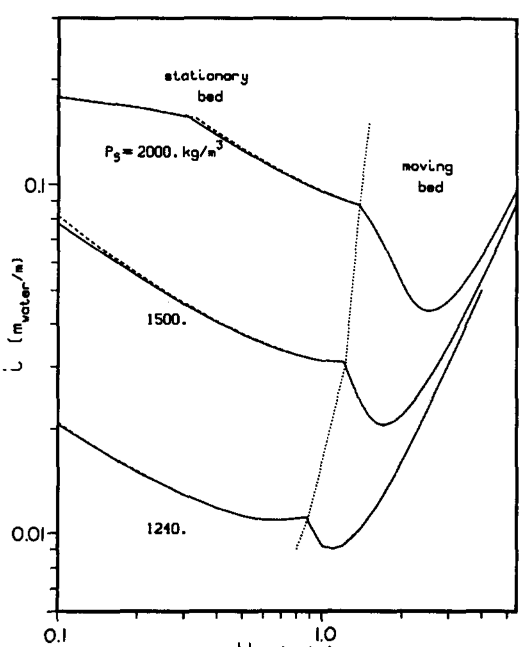

Figure 4. Dependence of pressure drop on slurry superficial velocity. $\rho_L = 1000\ \mathrm{kg/m^3}$, $C_s = 5\%$, $d_p = 1$ mm, $D = 200$ mm; ——, original model; – – –, modified model.

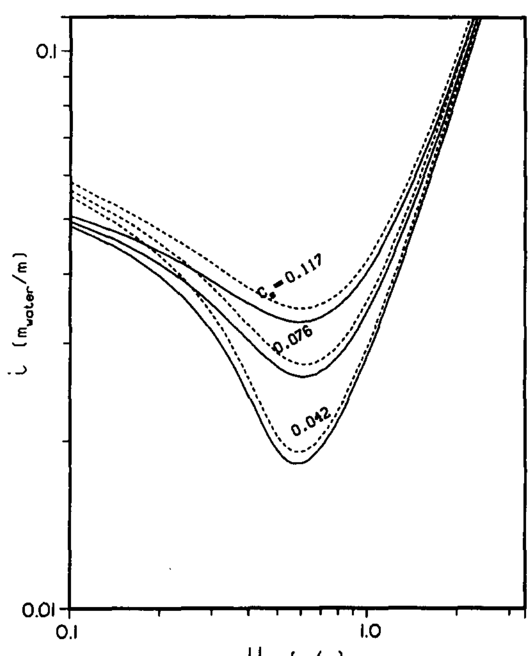

Figure 5. Dependence of pressure drop on slurry superficial velocity. $\rho_S = 1240\ \mathrm{kg/m^3}$, $\rho_L = 1000\ \mathrm{kg/m^3}$, $d_p = 3$ mm, $D = 50$ mm; ——, original model; – – –, modified model (constants B_1 and B_2 [11a,b] divided by 20).

Various sets of operational conditions (in which system parameters such as particle and pipe diameters, solids density and mixture concentration were varied over a wide range) were tested and the results show the same tendencies for all of them.

CONCLUSION

It has been shown that the assumption of no-slip in the bed layer used by Doron *et al.* (1987) is a valid approximation. It applies to the pressure drop predictions in both the moving bed and the stationary bed flow patterns as well as to the calculation of the mixture velocity at the transition between them. Thus, the previous no-slip assumption is justified as it simplifies the model and reduces computation time considerably.

A slight modification of the previous model is suggested here. That is, to use the liquid density, ρ_L, instead of ρ_b in [14]. The difference in the results is small, yet the latter allows consistency between the previous simplified model and the modified one and is physically reasonable.

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